

# Investigation of Top quark spin correlations at hadron colliders

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## Abstract

We report on our results about hadronic  $t\bar{t}$  production at NLO QCD including  $t, \bar{t}$  spin effects, especially on  $t\bar{t}$  spin correlations.

## 1. Introduction

Top quarks, once they are produced in sufficiently large numbers, are a sensitive probe of the fundamental interactions at high energies. On the theoretical side this requires precise predictions, especially within the Standard Model (SM). As far as  $t\bar{t}$  production at the Tevatron and LHC is concerned, spin-averaged (differential) cross section have been computed at next-to-leading order (NLO) QCD [1,2] including resummations [3,4]. Observables involving the spin of the top quark can also be calculated perturbatively, especially within QCD. It is expected that such quantities will play an important role in exploring the interactions that are involved in top quark production and decay. Within the SM, the QCD-induced correlations between  $t$  and  $\bar{t}$  spins are large and can be studied at both the Tevatron

and the LHC, for instance by means of double differential angular distributions of  $t\bar{t}$  decay products. Results for these distributions at NLO QCD [5–8] are reviewed below.

## 2. Theoretical Framework

We consider the following processes at hadron colliders

$$pp/p\bar{p} \rightarrow t\bar{t} + X \rightarrow \begin{cases} l^+ l'^- & + X \\ l^\pm j_{t(\bar{t})} & + X \\ j_t j_{\bar{t}} & + X \end{cases} \quad (1)$$

where  $l$  ( $l'$ ) =  $e, \mu, \tau$ , and  $j_t$  ( $j_{\bar{t}}$ ) denotes the jet originating from non-leptonic  $t$  ( $\bar{t}$ ) decay. At NLO QCD the following parton reactions contribute to the above processes:

$$\begin{aligned} gg, q\bar{q} &\xrightarrow{t\bar{t}} b\bar{b} + 4f, \\ gg, q\bar{q} &\xrightarrow{t\bar{t}} b\bar{b} + 4f + g, \\ g + q(\bar{q}) &\xrightarrow{t\bar{t}} b\bar{b} + 4f + q(\bar{q}), \end{aligned} \quad (2)$$

where  $f = q, \ell, \nu_\ell$ . The calculation of these cross sections at NLO QCD simplifies in the leading pole approximation (LPA), which is justified because  $\Gamma_t/m_t < 1\%$ . Within the LPA, the radiative corrections can be classified into factorizable and non-factorizable contributions. At NLO these non-factorizable corrections do not contribute to the double differential distributions [8], which we will discuss below. Therefore we do not consider them here. Considering only factorizable corrections in the on-shell approximation the squared matrix element  $|\mathcal{M}|^2$  of the respective parton reaction is of the form

$$|\mathcal{M}|^2 \propto \text{Tr} [\rho R \bar{\rho}] = \rho_{\alpha'\alpha} R_{\alpha\alpha',\beta\beta'} \bar{\rho}_{\beta\beta'}. \quad (3)$$

Here  $R$  denotes the density matrix that describes the production of on-shell  $t\bar{t}$  pairs in a specific spin configuration, and  $\rho, \bar{\rho}$  are the density matrices describing the decay of polarized  $t$  and  $\bar{t}$  quarks, respectively, into specific final states. The subscripts in (3) denote the  $t, \bar{t}$  spin indices. The spin-averaged production density matrices yield the NLO cross sections for  $t\bar{t}$  being produced by  $q\bar{q}$ ,  $gg$ ,  $gq$ , and  $g\bar{q}$  fusion [1, 2].

To obtain a full NLO QCD analysis of (1), we must consider also the NLO QCD corrections to the matrix element of the main SM decay modes of the (anti)top quark in a given spin state, i.e. the semileptonic modes  $t \rightarrow b\ell^+\nu_\ell$ ,  $b\ell^+\nu_\ell g$  ( $\ell = e, \mu, \tau$ ), and the non-leptonic decays  $t \rightarrow bq\bar{q}'$ ,  $bq\bar{q}'g$  where  $q\bar{q}' = u\bar{d}, c\bar{s}$  for the dominant channels. For the computation of the double angular distributions (6), the matrix elements of the 2-particle inclusive parton reactions  $i \xrightarrow{t\bar{t}} a + b + X$  are required. Here  $a, b$  denote a lepton or a jet. In the LPA this involves the 1-particle inclusive  $t$  decay density matrix  $2\rho_{\alpha'\alpha}^{t \rightarrow a} = \Gamma^{(1)}(11 + \kappa_a \tau \cdot \hat{\mathbf{q}}_1)_{\alpha'\alpha}$ , where  $\hat{\mathbf{q}}_1$  is the direction of flight in the  $t$  rest frame and  $\Gamma^{(1)}$  is the partial width of the respective decay channel. An analogous formula holds for  $\bar{t}$  decay. The factor  $\kappa_a$  is the  $t$  spin analysing power of particle/jet  $a$ . Its value is crucial for the experimental determination of top spin effects, in particular of  $t\bar{t}$  spin correlations. For the standard  $V - A$  charged-current interactions these coefficients are known to order  $\alpha_s$  for semileptonic [9] and non-leptonic [10] modes. The charged lepton is a perfect analyser of the top quark spin, which is due to the fact that  $\kappa_\ell = 1 - 0.015\alpha_s$ . In the case of hadronic top quark decays, the spin analysing power of jets can be defined, for example

$$\kappa_b = -0.408 \times (1 - 0.340\alpha_s) = -0.393, \quad (4)$$

$$\kappa_j = +0.510 \times (1 - 0.654\alpha_s) = +0.474. \quad (5)$$

Here  $\kappa_b$  is the analysing power of the  $b$  jet and  $\kappa_j$  refers to the least energetic non- $b$ -quark jet defined by the Durham algorithm. Obviously the spin analysing power is decreased if one uses the hadronic final states to analyse the spins of  $t$  and/or  $\bar{t}$ . However, this is (over)compensated by the gain in statistics and by the efficiency with which the  $t$  ( $\bar{t}$ ) rest frames can be reconstructed.

With the above building blocks, we can discuss the following double angular distributions<sup>1</sup>

$$\frac{1}{\sigma} \frac{d\sigma}{d\cos\theta_1 d\cos\theta_2} = \frac{1}{4} (1 - C \cos\theta_1 \cos\theta_2), \quad (6)$$

where  $C$  is a measure of the  $t\bar{t}$  correlations;  $\theta_1$  ( $\theta_2$ ) is the angle between the direction of flight of particle/jet  $a_1$  ( $a_2$ ) in the  $t$  ( $\bar{t}$ ) rest frame with respect to reference directions  $\hat{\mathbf{a}}$  ( $\hat{\mathbf{b}}$ ), which will be specified below. For the factorizable corrections the exact formula  $C = \kappa_{a_1} \kappa_{a_2} D$  holds [7]. Here  $D$  is the  $t\bar{t}$  double spin asymmetry

$$D = \frac{N(\uparrow\uparrow) + N(\downarrow\downarrow) - N(\uparrow\downarrow) - N(\downarrow\uparrow)}{N(\uparrow\uparrow) + N(\downarrow\downarrow) + N(\uparrow\downarrow) + N(\downarrow\uparrow)}, \quad (7)$$

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<sup>1</sup>QCD-generated absorptive parts in the parton scattering amplitudes induce a small  $t$  and  $\bar{t}$  polarization, which to order  $\alpha_s^3$  is normal to the  $q\bar{q}, gg \rightarrow t\bar{t}$  scattering planes [12, 13].

where  $N(\uparrow\uparrow)$  denotes the number of  $t\bar{t}$  pairs with  $t$  ( $\bar{t}$ ) spin parallel to the reference axis  $\hat{\mathbf{a}}$  ( $\hat{\mathbf{b}}$ ), etc. Thus  $\hat{\mathbf{a}}$  and  $\hat{\mathbf{b}}$  can be identified with the quantization axes of the  $t$  and  $\bar{t}$  spins, respectively, and  $D$  directly reflects the strength of the correlation between the  $t$  and  $\bar{t}$  spins for the chosen axes.

For  $t\bar{t}$  production at the Tevatron it is well known that the so-called off-diagonal basis [11], which is defined by the requirement that  $\hat{\sigma}(\uparrow\downarrow) = \hat{\sigma}(\downarrow\uparrow) = 0$  for the process  $q\bar{q} \rightarrow t\bar{t}$  at tree level, yields a large coefficient  $D$ . It has been shown in [7] that the beam basis, where  $\hat{\mathbf{a}}$  and  $\hat{\mathbf{b}}$  are identified with the hadronic beam axis, is practically as good as the off-diagonal basis. A further possibility is the helicity basis, which is a good choice for the LHC.

### 3. Predictions for the Tevatron and the LHC

We now discuss the spin correlation coefficients  $C$  of the distributions (6). It should be noted that beyond LO QCD, it is important to construct infrared and collinear safe observables at parton level. In the case at hand it boils down to the question of the frame in which the reference directions  $\hat{\mathbf{a}}$  and  $\hat{\mathbf{b}}$  are to be defined. It has been shown that, apart from the  $t$  and  $\bar{t}$  rest frames, the  $t\bar{t}$  zero momentum frame (ZMF) is the appropriate frame for defining collinear safe spin-momentum observables. The off-diagonal, beam, and helicity bases are defined in the  $t\bar{t}$  ZMF. Details can be found in Ref. [8].

In Table 1 we list our predictions for  $C$  in (6) at the Tevatron and LHC. The results are obtained using the CTEQ6L (LO) and CTEQ6.1M (NLO) parton distribution functions (PDF) [14]. Numbers are given for the dilepton (L-L), lepton+jet (L-J) and all-hadronic (J-J) decay modes of the  $t\bar{t}$  pair, in the latter two cases the least energetic non- $b$ -quark jet (defined by the Durham cluster algorithm) was used as spin analyser. One notices that for the Tevatron the spin correlations are largest in the beam and off-diagonal bases, and the QCD corrections reduce the LO results for the coefficients  $C$  by about 10% to 30%. For the LHC the QCD corrections are small ( $< 10\%$ ). These results are obtained with  $\mu \equiv \mu_R = \mu_F = m_t = 175$  GeV. At the Tevatron, a variation of the scale  $\mu$  between  $m_t/2$  and  $2m_t$  changes the results at  $\mu = m_t$  by  $\sim \pm (5-10)\%$ , while at the LHC the change of  $C_{\text{hel}}$  is less than a per cent.

In Table 2 we compare the NLO results for the spin correlation coefficients evaluated for the CTEQ6.1M, MRST2003 [15] and GRV [16] PDFs. It is easy to see that the results with the recent CTEQ6.1M and MRST2003 PDF agree at the per cent level (this is not the case for previous versions of the CTEQ and MRST PDF),

Table 1: *LO and NLO results for the spin correlation coefficients  $C$  of the distributions (6) for the Tevatron at  $\sqrt{s} = 1.96$  TeV and for LHC at  $\sqrt{s} = 14$  TeV. The PDF CTEQ6L (LO) and CTEQ6.1M (NLO) were used, and  $\mu_F = \mu_R = m_t = 175$  GeV.*

|                   |     | L-L    | L-J    | J-J    |
|-------------------|-----|--------|--------|--------|
| Tevatron          |     |        |        |        |
| $C_{\text{hel}}$  | LO  | -0.471 | -0.240 | -0.123 |
|                   | NLO | -0.352 | -0.168 | -0.080 |
| $C_{\text{beam}}$ | LO  | 0.928  | 0.474  | 0.242  |
|                   | NLO | 0.777  | 0.370  | 0.176  |
| $C_{\text{off}}$  | LO  | 0.937  | 0.478  | 0.244  |
|                   | NLO | 0.782  | 0.372  | 0.177  |
| LHC               |     |        |        |        |
| $C_{\text{hel}}$  | LO  | 0.319  | 0.163  | 0.083  |
|                   | NLO | 0.326  | 0.158  | 0.076  |

while the GRV98 PDF gives significantly different results at the Tevatron. This shows that the spin correlations are very sensitive to the relative quark and gluon contents of the proton [7]. Future measurements of (6) may offer the possibility to further constrain the quark and gluon contents of the proton.

Before closing this section, we summarize how an experimental measurement of the distributions (6) that matches our predictions should proceed: 1) Reconstruct the top and antitop 4-momenta in the laboratory frame (= c.m. frame of the colliding hadrons). 2) Perform a rotation-free boost from the laboratory frame to the  $t\bar{t}$  ZMF. Compute  $\hat{\mathbf{a}}$  and  $\hat{\mathbf{b}}$  in that frame. 3) Perform rotation-free boosts from the  $t\bar{t}$  ZMF to the  $t$  and  $\bar{t}$  quark rest frames. Compute the direction  $\hat{\mathbf{q}}_1$  ( $\hat{\mathbf{q}}_2$ ) of the  $t$  ( $\bar{t}$ ) decay product  $a_1$  ( $a_2$ ) in the  $t$  ( $\bar{t}$ ) rest frame. Finally, compute  $\cos\theta_1 = \hat{\mathbf{a}} \cdot \hat{\mathbf{q}}_1$ ,  $\cos\theta_2 = \hat{\mathbf{b}} \cdot \hat{\mathbf{q}}_2$ . Here one should notice that in this prescription the  $t$  and  $\bar{t}$  rest frames are obtained by first boosting into the  $t\bar{t}$  ZMF. If this step is left out, and the  $t$  and  $\bar{t}$  rest frames are constructed by directly boosting from the lab frame, a Wigner rotation has to be taken into account.

Table 2: *Spin correlation coefficients at NLO for different PDFs for the Tevatron (upper part) and the LHC (lower part) for dilepton final states.*

| Tevatron          |          |          |        |
|-------------------|----------|----------|--------|
|                   | CTEQ6.1M | MRST2003 | GRV98  |
| $C_{\text{hel}}$  | -0.352   | -0.352   | -0.313 |
| $C_{\text{beam}}$ | 0.777    | 0.777    | 0.732  |
| $C_{\text{off}}$  | 0.782    | 0.782    | 0.736  |
| LHC               |          |          |        |
| $C_{\text{hel}}$  | 0.326    | 0.327    | 0.339  |

## 4. Conclusion

We have computed at NLO QCD the  $t\bar{t}$  spin correlations in hadronic top production, which are large effects within the SM. Our present results are obtained without imposing kinematic cuts. Such cuts will in general distort the distributions, i.e.  $C$  will in general depend on the angles  $\theta_1$  and  $\theta_2$ . One strategy is to correct for these distortions by Monte Carlo methods before extracting the spin correlation coefficient and comparing it with theoretical predictions. A future aim is to directly include the cuts in an NLO event generator to be constructed with our NLO results for all relevant  $2 \rightarrow 6$  and  $2 \rightarrow 7$  processes.

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